Musical kaleidocycles: composition and analytical techniques

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1. Introduction

Many composers of the 20th century used cyclic harmonic patterns, although they did not develop a unitary theory to systematize their use. An analysis of these patterns is particularly interesting as regards composition: the purpose of this article is to show how a reset of cyclic elements inherited from the tradition could be a starting point for a kaleidocyclic technique in musical composition. At the beginning of the 20th century, several composers were fascinated by cyclic chord relations, mainly at major/minor third, tritone and augmented triad. These relations allowed to break definitely with the tonal system, based on the fifth-relation, and to explore the features of interval relations dividing the octave into equal parts. Thus, specific sections of works based on such harmonic modules (Ravel, Stravinskij, Szimanowsky), two-part pieces with tritone repeat (Skrjabin) and also musical fugues with entries at third or tritone (Bartok) were composed. There were not strict cycles yet, but those works showed a tendency to periodical repetition of elements upon planned transposition levels, ending with the initial position, similarly to static progressions. From a historical point of view, the earliest general theories on the octave division into equal parts date back to the middle of the 19th century, but the topic started to be dealt with more systematically only at the beginning of the 20th century, with the disruption of the tonal system, and it was only at the beginning of the 1960s that its manifold implications started to be analyzed deeply, especially thanks to the techniques developed in the field of the American Set-theory, but not only.

Italian Domenico Alaleona stands out as one of the first theorists to highlight this topic: he pointed out that in the tonal system a chord dividing the octave into equal parts can belong to as many tonalities as the sounds composing it [Alaleona, 1911]. The characteristic of these chords, which Alaleona names "neutral chords", is their static nature, lack of direction, suspension, ambiguity. This definition recalls the theory of "dual rhythm", worked out during the same years by the Ukranian musicologist Boleslav Javorskij; in the field of music, this theory was fully applied in Skrjabin's late works [Cholopov, 1975]. Some of the most relevant theorists of the 20th century, as Slonimskij, Perle and Lendvai, developed their theories on this point. In The perfect pitch, Slonimskij writes as follows:

> The antiphonal strength of modulatory processes and fugal imitation had its source in the unequal division of the octave into two parts, from the tonic to the dominant and from the dominant to the tonic, leading to non-symmetric procedures. But why not try a democratìc division of the octave in two equal parts, with the tritone rather than the perfect fifth as theline of demarcation? [1988: 173].

Antokoletz works out the same concepts when, in a study on the cyclic intervals in the Russian ballets, he writes:

> In general, the system of interval cycles can be outlined according to certain basic principles of intervallic construction. Each pair of complementary intervals — unison/octave, minor second/major seventh, and so on, through tritone/tritone — consists of two intervallic differences that add up to an octave. In the pairs of complementary intervals other than that of the perfect fourth/perfect fifth, the smaller interval of each pair generates a cycle that subdivides one octave into equal parts (The perfect fourth is unique in that it generates a cycle of all twelve tones

through many octaves before thè initial pitch class returns). Thus, there is one cycle of semitones, two of whole tones, three of minor thirds, four of major thirds, only one of perfect fourths, and six of tritones [1986: 579].

Musicologist Erno Lendvai dealt specifically with the use of cyclic schemes in Bartok (1971): he asserted that in Bartok's system the three traditional functions of tonic, dominant and subdominant could be expressed by three diminished sevenths, each of which to be intended not as a chord but as a functional relation among four different tonalities. Then, each function would have four "poles" set along the axis of a diminished seventh: tonic (C, E flat, F sharp,A), dominant (E, G, B flat, C sharp), subdominant (A flat, B, D, F). According to Lendvai, this system was made necessary by a historical need, represented by the logical continuation of European functional music, reflecting the contraposition of the principles of tonality and those of equivalence, with the gradual predominance of the latter up to the free and equal treatment of the twelve chromatic notes.

2. Common-notes numerical vector

In dealing with this topic, I would like to start with some personal experiences. Since my studies at the Conservatoire, I have been interested in the possibilities of linking a chord to itself. Manfred Kelkel's original studies on Aleksandr Skrjabin are an important reference point [Kelkel 1984].

Aleksandr Skrjabin's late works are sometimes arranged on a continuously transformed single chord: in particular, Skrjabin's Prometheus is totally based on one chord (known also as "mystic chord" because of its typical sound); this chord mostly links itself with its transpositions at an ascending/descending-minor-third or tritone distance. In practice, all chords placed upon the axis of a diminished-seventh can be considered as belonging to the same group, as they link one with the other much more frequently than with the others [Verdi 1996, 66-76].

Every vertical combination of notes inside the chromatic scale (chord) can be transposed twelve times, and has a more or less close relation with its own transpositions, in connection with the common notes it shares with them. All transpositions of a particular combination are defined as different representations corresponding to the same form. The total amount of transpositions of a combination is defined as set of equivalent combinations by transposition [Verdi 1998, 38-41].

In surveying the various possibilities to link any chord to its transpositions, it is possible to identify actual modal groups typical of every chord, depending on the amount of notes in common with the transpositions (common-notes numerical vector). The vector is fundamental to a good understanding of some properties of single combinations, and stands as a sort of "genetic code" containing all necessary indications to identify particular relations. The vector can be identified by adopting some reference values, according to a process which has been standardized during the twentieth century, through the contribution of several theoretical studies [see Verdi 2007]. If a fundamental pitch of the chord (=0) is given, a set of numbers indicates all pitches, considering a semitone as corresponding to 1. Thus, the Prometheus chord (C, C sharp, D sharp, F, G, A) corresponds to (0, 1, 3, 5, 7, 9) and its interval structure – the series of semitones separating every note of the chord - is (1-2-2-2-2-(3), where the last number in brackets indicates the interval required to complete the octave. The sum of digits of the interval structure must be equal at 12.

The number of notes which are shared by the various chord transpositions (vector) can be calculated empirically by means of a simple system of Cartesian axis reporting all 12 transpositions arranged one upon the other. The arrangement of common notes upon various transposition levels - which can be compared to some pieces arranged on a draught-board (on the abscissa there are lined the chord sounds, on the ordinate the common sounds upon 12 transposition levels) – is evidently seen on the chart (Fig.1), based on Skrjabin’s “Prometheus chord” (0,1,3,5,7,9).

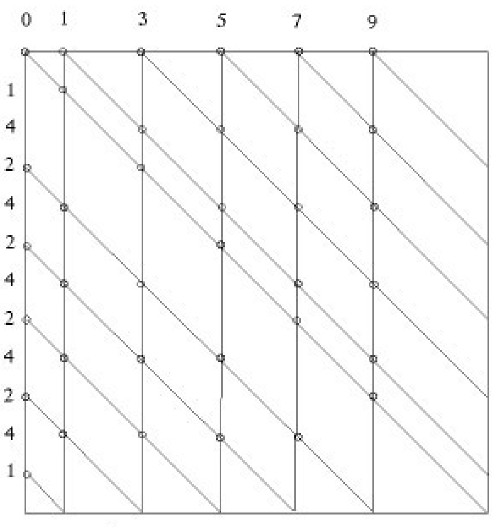


Fig. 1

The previous “transposition diagram” [Verdi 2005, 203-222] can also be represented by the following chart, where common notes shared by a chord and its transpositions are highlighted (See Tab.1)

Chromatic scale	0	1	2	3	4	5	6	7	8	9	10	11	Vector
Chord	o	o		o		o		o		o			
0	**o**	**o**		**o**		**o**		**o**		**o**			6
1		**o**	o		o		o		o		o		1
2			o	**o**		**o**		**o**		**o**		o	4
3	**o**			**o**	o		o		o		o		2
4		**o**			o	**o**		**o**		**o**		o	4
5	**o**		o			**o**	o		o		o		2
6		**o**		**o**			o	**o**		**o**		o	4
7	**o**		o		o			**o**	o		o		2
8		**o**		**o**		**o**			o	**o**		o	4
9	**o**		o		o		o			**o**	o		2
10		**o**		**o**		**o**		**o**			o	o	4
11	**o**		o		o		o		o			o	1
Transposition level													

Tab. 1

The common-notes numerical vector indicates the elements shared by a chord and its transpositions; it can be expressed symbolically in various ways: in its complete form is made up of 12 digits, referred to as 'entrees', each of which corresponds to the notes shared by a combination and its 11 transpositions. In the previous case we have: (6)14242 4 24241. In essence, in all its 12 transpositions the Prometheus chord (0,1,3,5,7,9) gives rise to three subsets consisting of those chords that share the same number of notes in common, namely 1, 2 and 4. (See Table n. 2).

A	B	C
1 nota in comune	2 note in comune	4 note in comune
0 1 3 5 7 9 1 2 4 6 8 10 0 2 6 8 11	0 1 3 5 7 9 0 3 4 6 8 10 0 2 5 6 8 10 0 2 4 7 8 10 0 2 4 6 9 10	0 1 3 5 7 9 2 3 5 7 9 11 1 4 5 7 9 11 1 3 7 8 9 11 1 3 5 8 9 11 1 3 5 7 10 11

Tab. 2

A graphical representation particularly useful, because it allows to immediately check the relationship between a chord and its transpositions, is shown in Fig n. 2. On top of a dodecagon that represents the total of 12 notes were inserted numbers corresponding to common notes among the original chord and its transpositions. In this way the two transpositions, which have one common note with the original, give rise, with the original itself, to an isosceles triangle; the four transpositions that have two notes in common with the original give rise to an irregular pentagon, while the 5 transpositions that have four notes in common with the original give rise to a regular hexagon [Verdi, 2005-223-28].

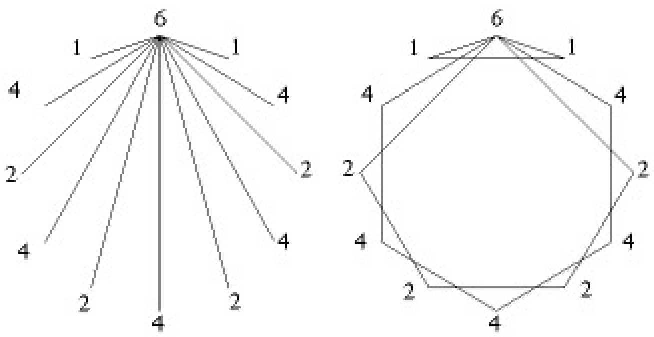


Fig. 2

In a particular case, the combinations, which reproduce themselves in full upon all transposition levels corresponding to their pitch and are built upon a unique interval repeating itself cyclically are: tritone (6), augmented-fifth (4), diminished-seventh (3), exatone-scale (2) and chromatic scale (1).

The example below (Fig. 3) represents the whole-tone chord (exatonal), which alternates with regularity 6 and 0 notes in common with the original one.

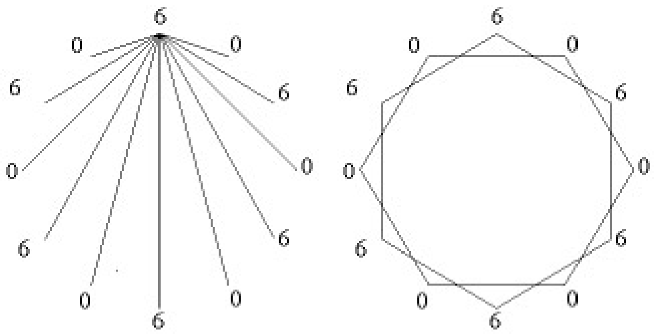


Fig. 3

The transposition diagrams of the numerical vector of common notes have varied and interesting geometrical arrangement properties. If each pitch corresponds to a particular colour (from c-red to b-violet), coloured diagrams can be derived. The graphs shown here (Fig. 4), originally watercolors on cardboard or chalk, are from a series of works that I developed in the Eighties.

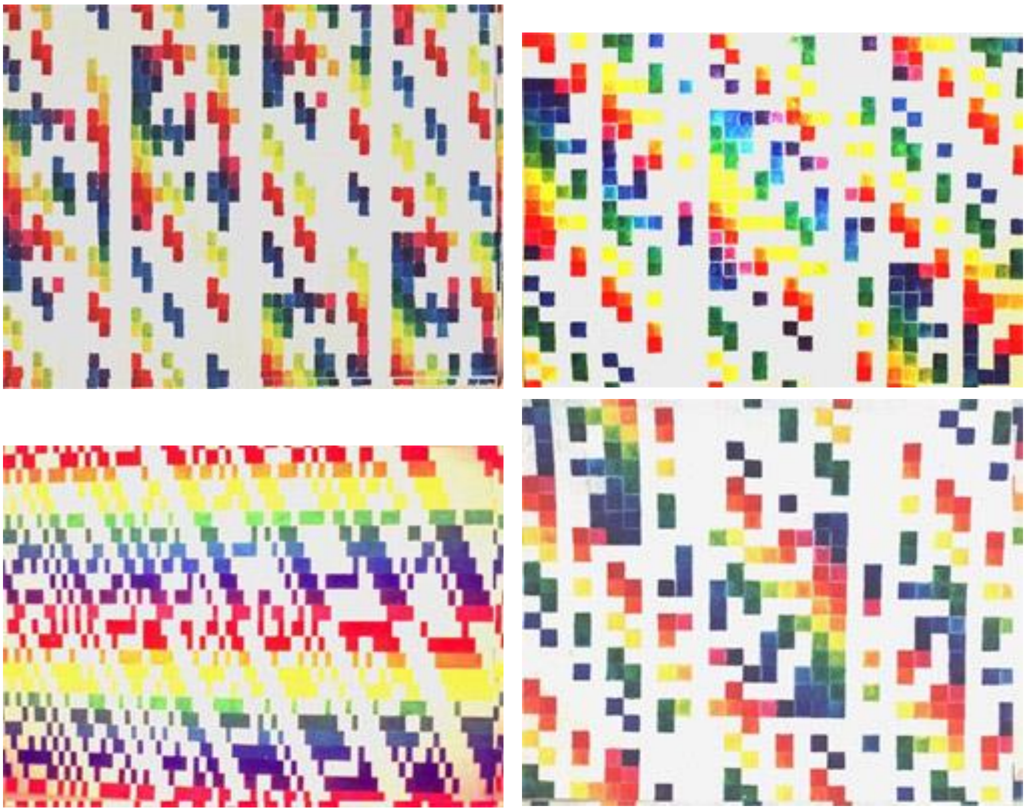


Fig. 4

3. Supplementary vector

A chord transposed upon its constitutive pitches forms a sort of metre-harmonic group, where horizontal and vertical sequences are regulated by the same rule. Besides, some notes are repeated more frequently than others. In the Prometheus chord case, there will be the following repetitions for every pitch [Verdi 2002].

In the Tab.3, the chord of Skrjabin's Prometheus (0,1,3,5,7,9) is placed upon grades 0,1,3,5,7,9, corresponding to its pitches at 1-2-2-2-2-3 semitone intervals. Consequently within the dodecagon representing the chromatic scale, the irregular hexagon generated by chord (0,1,3,5,7,9), transposed on 013579 levels, creates a figure of fractal type, as each agreement generates levels of transpositions that in turn generate the same chord, and so on (Fig. 5).

Chromatic scale	0	1	2	3	4	5	6	7	8	9	10	11	
Chord	o	o		o		o		o		o			
0	o	o		o		o		o		o			
1		o	o		o		o		o		o		
2													
3	o			o	o		o		o		o		
4													
5	o		o			o	o		o		o		
6													
7	o		o		o			o	o		o		
8													
9	o		o		o		o			o	o		
10													
11													
	5	2	4	2	4	2	4	2	4	2	5	0	Supplementary vector
Transposition levels													

Tab. 3

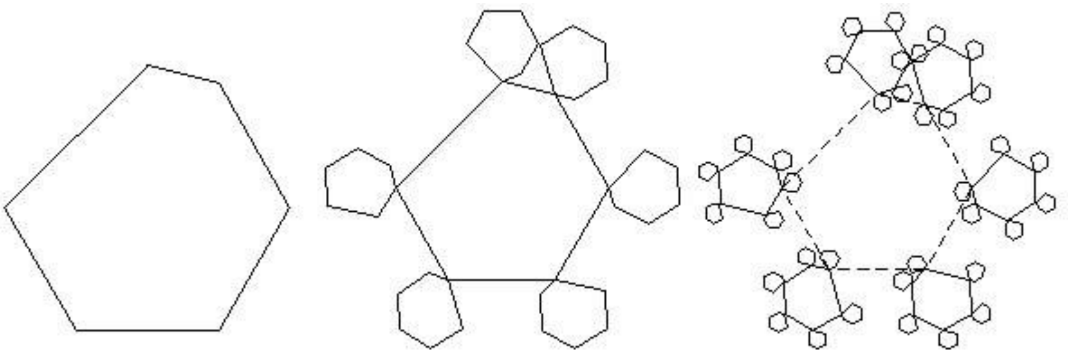


Fig. 5

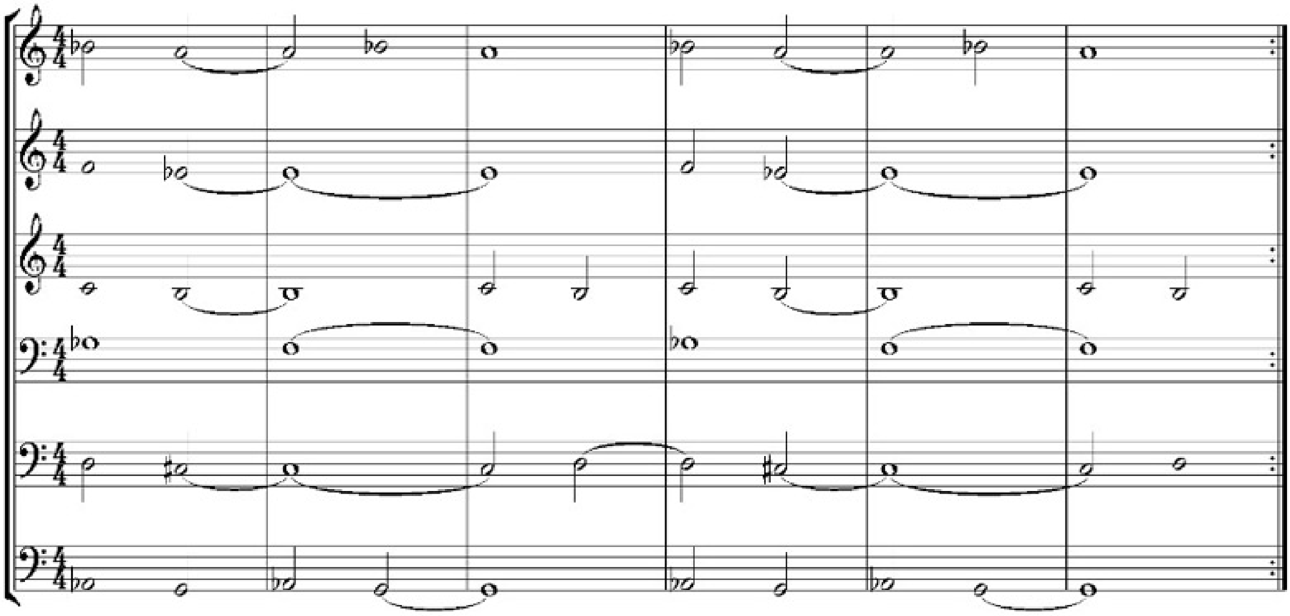
The series of number indicating the sets of common notes of a chord if transposed upon its pitches – in this case 524242424250 – is named supplementary vector and has some interesting properties.

Infact, every entry of the supplementary vector (sv) which is summed to the entries corresponding to the common-note-numerical vector of the chords gives 6 as a result.

supplementary vector entries	052424242425
common-note vector entries	614242424241

	666666666666

The previous chart has an immediate and effective practical application. The following musical example (Ex. n.1) is built on the Prometheus chord transposed on levels corresponding to his pitches; in Fig n. 6 the musical result is displayed through the use of colors.



Ex. 1

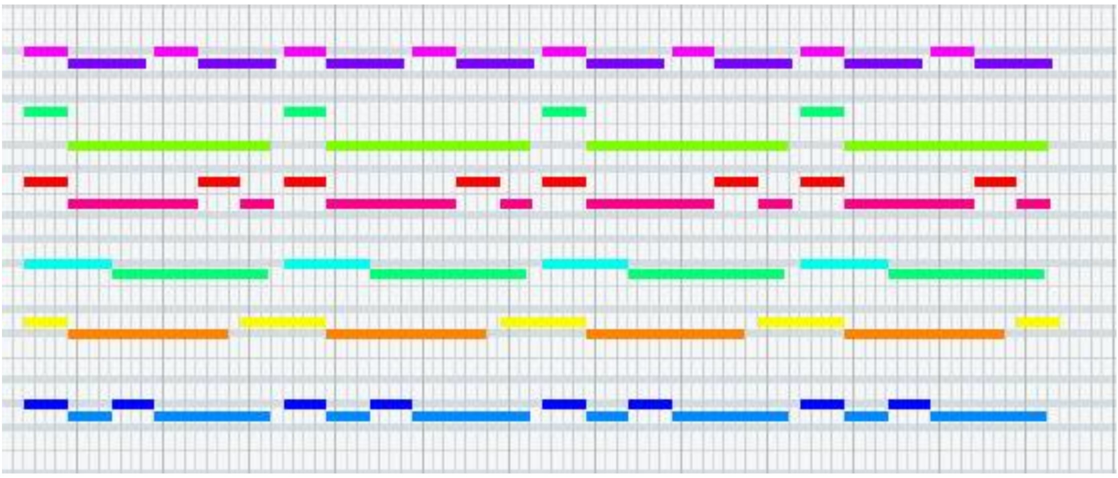


Fig. 6

- Audio sample 1 -

4. Theory of cycles

In the twelve-tone system, every group of intervals can be graphed by a set of segments inscribed in a regular dodecagon. The set of all possible intervals may be represented by a figure composed of 66 segments (Fig. n.7)

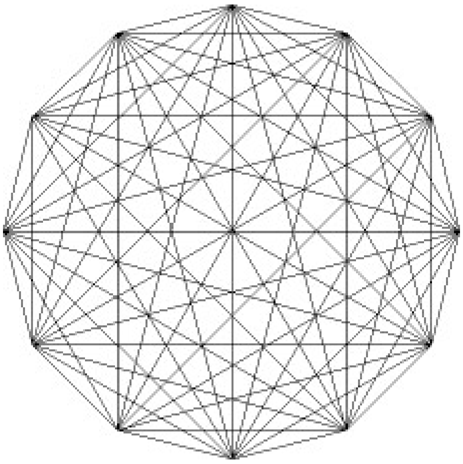
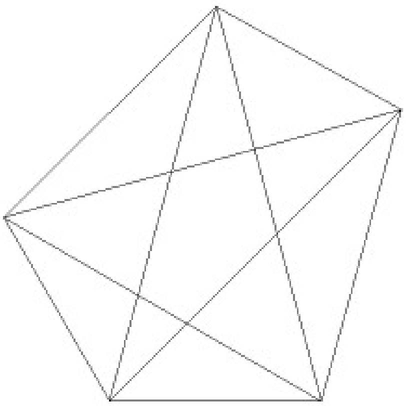


Fig. 7

Any group of intervals may be represented by a portion of the main figure; then all the music written upon the twelve-note chromatic scale may be reduced to a sequence of chords related one another upon intervals drawn from the main figure. Tonal music, for instance, is mainly structured on relations involving I, V, IV, VI and II degree, as shown in Fig n. 8.



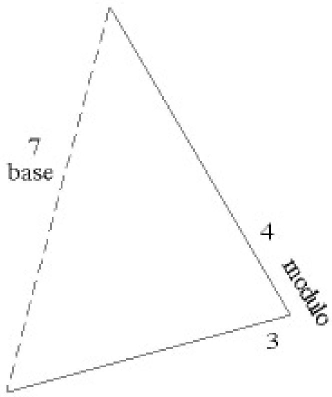


Fig. 8 and 9

A link based upon groups of intervals periodically repeating themselves upon the various pitches of the chromatic scale originates a cycle. The group of intervals repeating itself is the module of the cycle. The module repeats itself upon a unit interval named base, given by the sum of the elements of the module. In Fig.9 is shown the case of module (4-3): the base amounts to 7 (as 4+3=7), and the subsequent repetitions of the module 4-3 are so placed upon interval 7 [Verdi 2005, 22].

Base values inferior to the octave (12) are on simple module, if they are superior, they are on compound module: since a base can assume only values from 0 to 11, other values may be reduced to an octave by one operation named mod(12). For instance a compound module 4-5-

6 is reduced to a base 15(mod12)=3.

The amount of intervals forming a module is named metre; in the previous case, the cycle upon module (4-3) is regular and metrically structured 2. Note that if the metre is=1, base and module are equivalent.

If a cycle completes itself within the first octave, it is a simple cycle; if, on the contrary, it is composed upon various octaves, it is a compound cycle. The unit of all necessary repetitions for a module to close its cycle is named period: the period is a number worked out by adding the base to itself, until forming a 12-multiple; from this number it is possible to check whether the cycle is simple or compound. In the Fig. 10 the module (3-4-2-3) of metre 4 is proposed: placed upon base 12, appears only once before going back to the initial level, giving rise to a period-1- simple cycle, since (3+4+2+3) = 12. A base amounting to 12 always gives rise to a simple cycle.

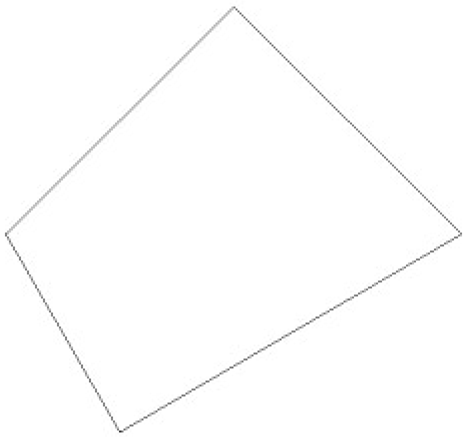


Fig. 10

The various pitches upon which the base repeats itself in order to complete a period give rise to a base class. The base class is the sequence of levels necessary to complete his inner articulation until the completion of the period; for exemple in the case of base 5, the base class will be 0,5,10,3,8,1,6,11,4,9,2,7. A consequence of this, for example, will be that the canons derived from kaleidocycles will set the voices entries necessarily at semitone, fourth or fifth and eleventh (base class 12), tone (base class 6), minor third (base class 4), major third (base class 3), tritone (base class 2). In the case of metre 2 module (4+3) the base 7 must repeat itself 12 times before it gets back to its original position, thus giving rise to a period-12-compound cycle; infact, base (4+3=7) is supposed to add up to itself 12 times in order to form a 12-multiple: (7x12=84).

A segment is the bidimensional projection of an interval. In the graphic representation of a module (4-3)-cycle, the module is represented by two segments (0-4)(4-7), while the complete cycle is represented by 24 segments meeting one another upon the 12 vertexes of the generating dodecagon.

In module (4-3) an interval between the same pitches never repeats itself before the end of the compound cycle; if transparency graphs are used, also with colours, reproducing a module by successive rotations it is possible to discover a figure including the whole compound cycle (See Fig. n. 11) [Verdi 2005, 181-193]

In the next example (Fig. n.12a) I tried to represent the various octaves on which the cycle takes place by moving each subsequent octave on a largest dodecagon. In this way one gets the impression of a two-dimensional moving of the reference polygon and that could reproduce graphically the effect due to the octave displacement. The same type of effect could be obtained with the colours, changing the color of the module 4-3 at every cyclic repetition (Fig.n.12b). The colours could also be used in different ways, for example to highlight the octaves needed to complete the cycle. Combining then the intervals of the module with the segmehomothetic, in which points situated on different plans are linked (Fig. n. 12c).

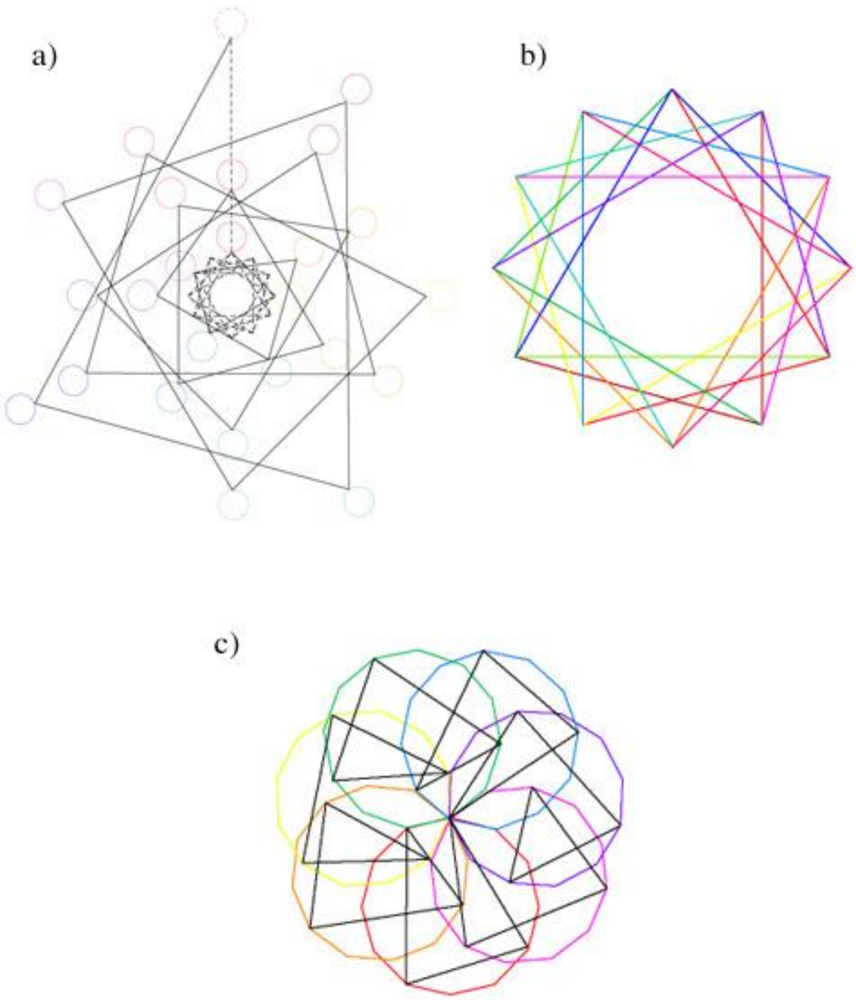


Fig. 12 - a, b, c

In a complete or incomplete compound cycle there may be repetitions of pitches and of intervals. The nature of repetitions is done by module and its derived period. A peculiar case is when a compound cycle completes itself without repetitions of pitches and of intervals (in this case it is named perfect). One such instance is shown in the following example (Fig. n. 13), upon module 3-7.

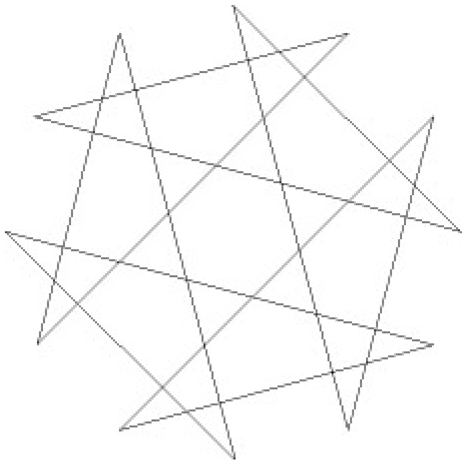


Fig. 13

It should be made clear that, starting from different modules, different cycles are obtained and hence is useful to know how many cycles can be made on modules of "n" components. The total amount of possible cycles for a given metre is said metrical class (mc) and it calculated by means of a simple summation operation: cm= b(b-1)(b-2)…(b-m+1)/m!.
In the case of metre 2, the metrical class will include 55 cycles, because 11x10/2x1=55 It is possible to demonstrate that the 55 metre-2-cycles give rise to 40 graphic representations, as 15 cycles are trivial. In the Tab. 4 follows the complete list of metre-2-cycles [Verdi 2006, 167-180].

	complete	**Complete**	**complete**	**incomplete**	**incomplete**
Simple	(1-1)			(1-2)(2-1) (1-3)(3-1) (1-5)(5-1) (2-4)(4-2)	(2-2) (3-3) (4-4)
Compound	(5-5)	(1-4)(4-1) (1-6)(6-1) (1-10)(10-1) (2-3)(3-2) (2-9)(9-2) (2-5)(5-2) (3-4)(4-3) (3-8)(8-3) (4-7)(7-4) (5-6)(6-5)	(1-9)(9-1) (3-7)(7-3)	(1-7)(7-1) (1-8)(8-1) (2-6)(6-2) (2-7)(7-2) (3-5)(5-3) (4-5)(5-4)	(2-8)(8-2) (3-6)(6-3) (4-6)(6-4)
	symmetrical	**symmetrical**	**asymmetrical**	**asymmetrical**	**symmetrical**

Tab. 4

5. Application of chords

Any chord may be applied to a cycle. At this stage it is necessary to introduce the concept of supplementary chord, meant as a set of notes which is applied to a cycle. A fundamental issue in order for this system to be consistent is defining the module which must be applied to a supplementary chord. The module can be chosen at random among the many possible ones, but it is convenient to establish a relation between module and chord. One first consistent step in this view can be establishing an analogy between a cycle-module and a chord-interval construction.

Considering now a module (2-3) spreading into a 24-interval-compound cycle, the development of the module gives rise to the following scheme (Tab. n. 5 e Fig. n. 14):

base module	(0, 2)	(5,7)	(10,0)	(3, 5)	(8,10)	(1,3)	(6,8)	(11,1)	(4, 6)	(9,11)	(2,4)	(7,9)
base class	(0)	(5)	(10)	(3)	(8)	(1)	(6)	(11)	(4)	(9)	(2)	(7)

module	2 – 3	2 - 3	2 - 3	2 - 3	2 - 3	2 - 3	2 - 3	2 - 3	2 - 3	2 - 3	2 - 3	2 - 3
base	5	5	5	5	5	5	5	5	5	5	5	5

Tab. 5

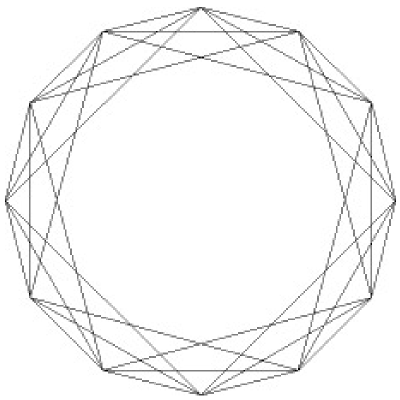


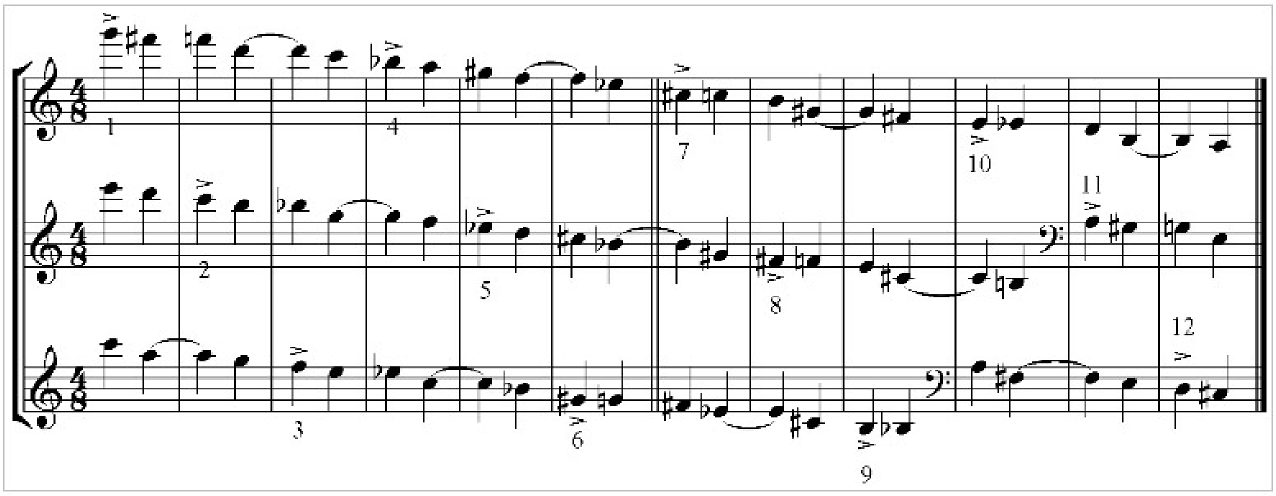
Fig. 14

As an example, a major chord can be applied to module 2-3 (0.4,7); this means that the chord is transposed cyclically, arranging its fundamental notes alternatively at a tone (2) and minor-third (3) distance. For the practical realization of the voice leading, it is useful to develop a graphic scheme, by transcribing every transposed chord vertically, according to a horizontal axis representing the base class. Spots represent single notes. The base class along the time axis is marked red (See Tab. n. 6).

In the musical development of supplementary chords, every module repetition gives rise to a canon entry upon the transposition levels generated by the base (in this case, 5). Since the base rules the entrance of the different canon voices, it is possible to infer that its period corresponds to the number of voices. For instance, a base repeating itself 12 times before ending the cycle gives rise to a 12-voice-canon. Voices follow one another along the time axis at a distance corresponding to the module metre (in this case, 2). In the following scheme, the entries are marked green, while in the musical sample are marked by numbers:

Transposition levels (vertical)																								
5			o.		o			o		o	o					o								
4	o					o											o.		o			o		o
3							o.		o			o		o	o					o				
2		o		o	o					o											o.		o	
1											o.		o			o		o	o					o
0	o.		o			o		o	o					o										
11				o											o.		o			o		o	o	
10					o.		o			O		o	o					o						
9		o	o					o											o.		o			o
8									o.		o			o		o	o					o		
7	o			o		o	o					o											o.	
6		o											o.		o			o		o	o			
Base class (horizontal)	0	2	5	7	10	0	3	5	8	10	1	3	6	8	11	1	4	6	9	11	2	4	7	9

Tab. 6



Ex. 2

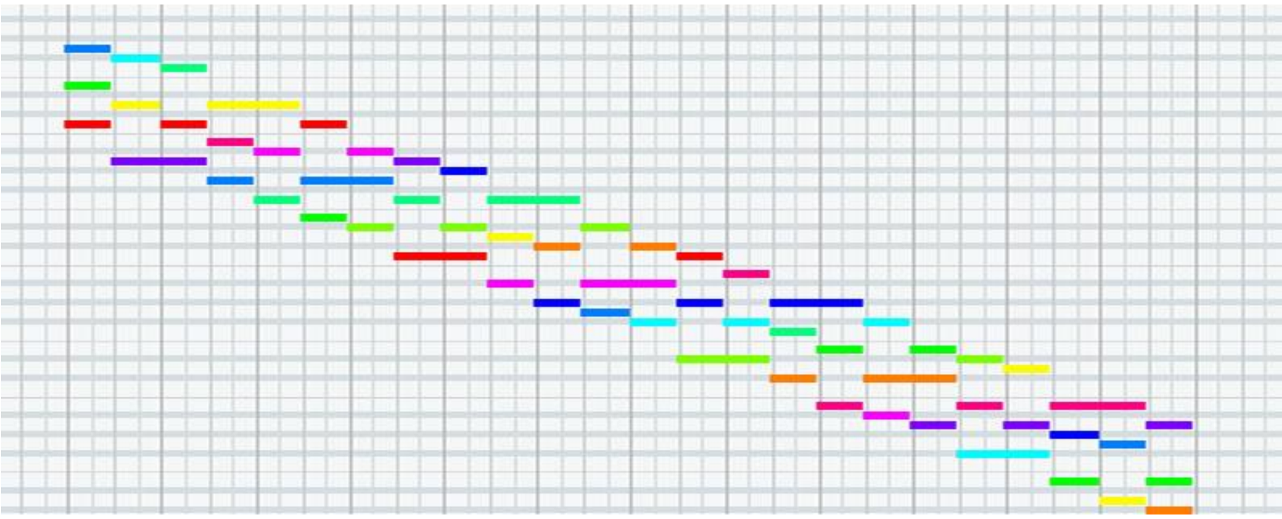


Fig. 15

- Audio sample 2 -

It is a 12-voice-rhythmic canon, where single entries are placed at metre-2-horizontal-distance and at base-5-vertical distance. The canon is infinite, the level upon which it starts (0) is merely conventional.

Thus a cyclical rhythmical scheme is generated by the vertical development of the chord upon its base, placed on the horizontal axis of the metre. The application of a supplementary chord to a cycle gives rise to a new original structure, which I have named as kaleidocycle, after some of Escher’s graphic techniques. The kaleidocycle has resulted in a transformation of space into time, that is a vertical structure which changes into a horizontal one.

It is possible to try and follow a methodical approach in the arrangement of modules deriving from the numerical vector of common notes of the chord used. If we want to define a metre-2-module linking the transposition of chord (0,1,3,5,7,9) alternatively upon its transpositions with 4 and 2 common notes on the basis of common-notes numerical vector, this is possible upon various ways, as from the following transposing grid (Tab. 16):

transp. level	0	1	2	3	4	5	6	7	8	9	10	11
cn vector	6	1	4	2	4	2	4	2	4	2	4	1

Tab. 16

We will get the cycle shown in Table n. 17.

module common notes	2 - 3 4-2	2 - 3 4-2	2 - 3 4-2	2 – 3 4-2	2 - 3 4-2	2 - 3 4-2	2 - 3 4-2	2 - 3 4-2	2 - 3 4-2	2 - 3 4-2	2 - 3 4-2	2 - 3 4-2
base	5	5	5	5	5	5	5	5	5	5	5	5

base module	(0, 2)	(5,7)	(10,0)	(3, 5)	(8,10)	(1,3)	(6,8)	(11,1)	(4, 6)	(9,11)	(2,4)	(7,9)
base class	(0)	(5)	(10)	(3)	(8)	(1)	(6)	(11)	(4)	(9)	(2)	(7)

Tab. 17

If the (0,1,3,5,7,9) chord is applied to this cycle, there derives a particularly consistent structure, since the cycle-module derives in turn from the chord. For example, in the following kaleidocycle (Ex. n. 3 and Fig. n. 16) the (0,1,3,5,7,9) chord transposed upon levels alternatively with 4 and 2 common notes generates a 6-entry-canon, with entries placed at ascending perfect fourth.

Ex. 3

This is the graphic representation of the previous sample:

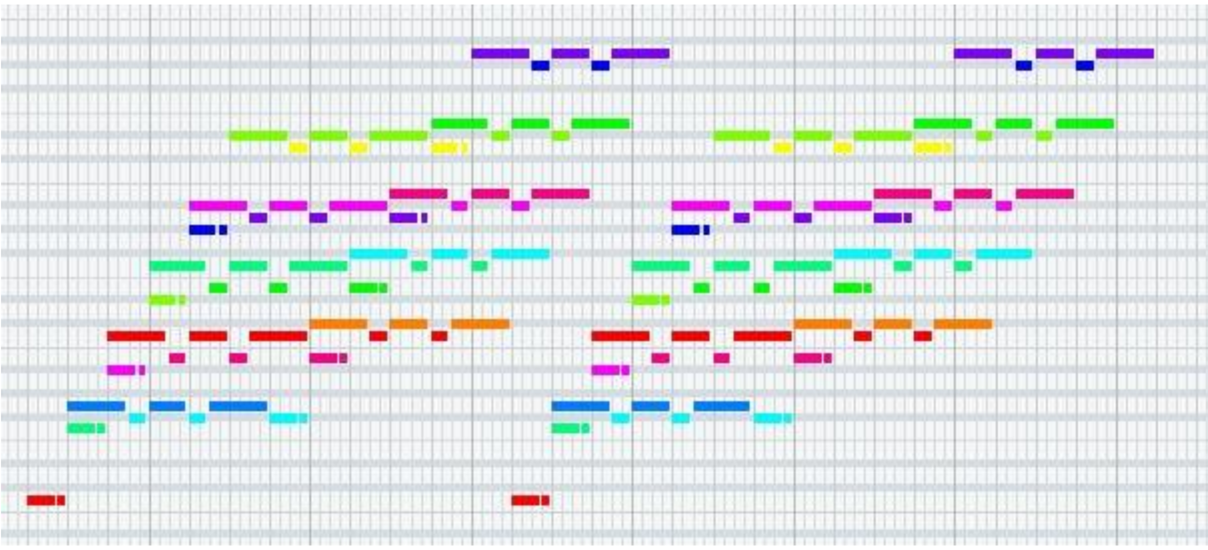


Fig. 16

- AUDIO sample 3 -

For the bidimensional graphic representation of kaleidocycles, it is necessary to work out a pattern of this kind, where spatial depth is obtained by means of a projection.

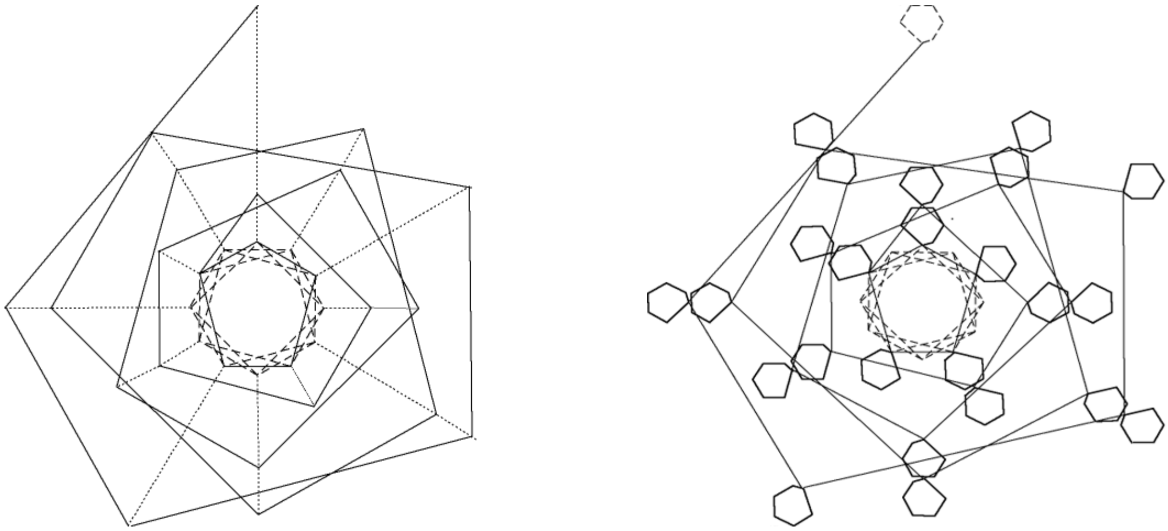


Fig. 17

The applied chord is set correspondingly to the transposition levels defined by the cycle. Then the graphic framework of the cycle can be cancelled. When highlighting the applied chord with different colours according to the transposition level, the following result is obtained (Fig. n. 18):

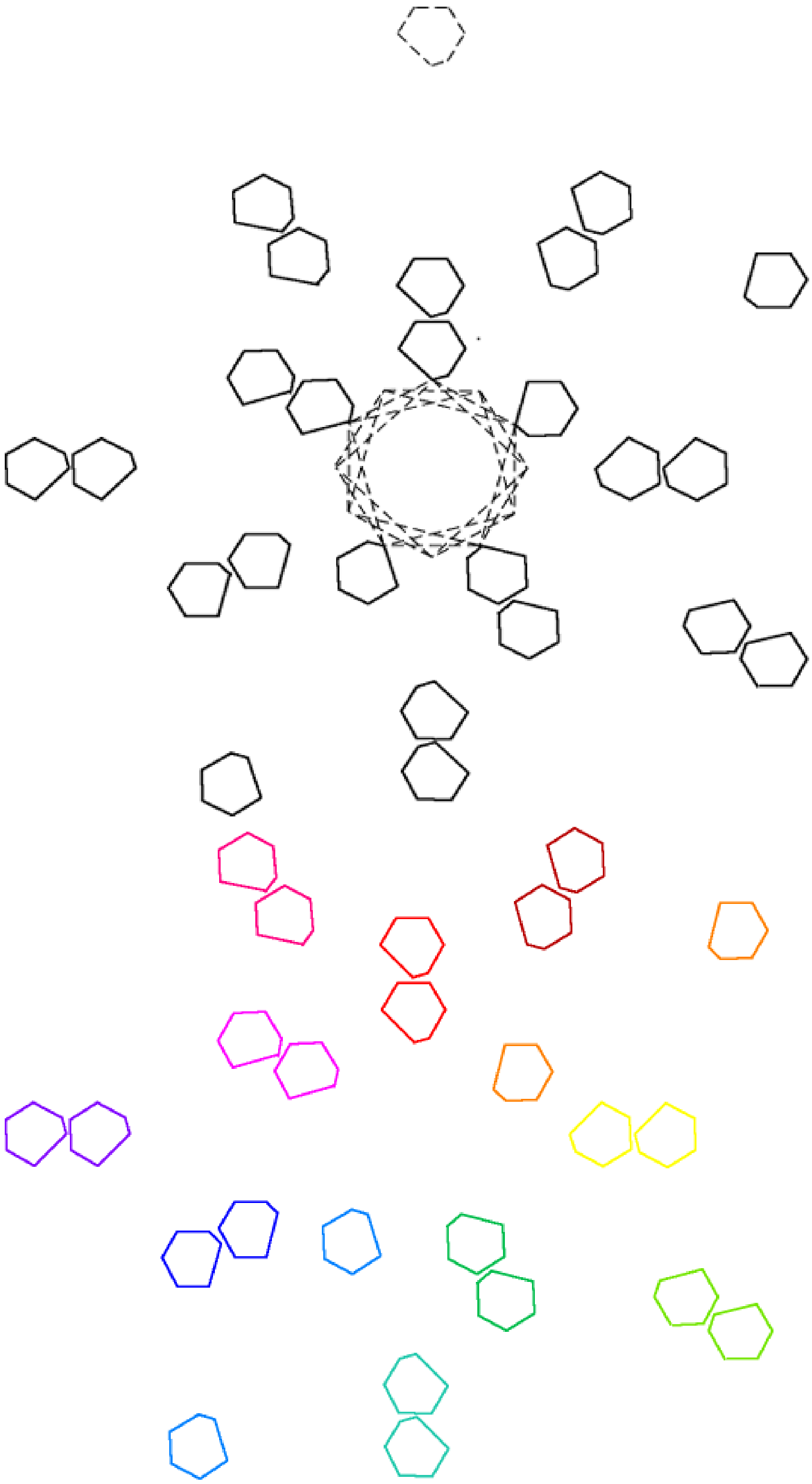


Fig. 18

For a correct spatial representation of a musical kaleidocycle, a four-dimensional space would be necessary, since in a three-dimensional representation the synchronous time elements of the structure are in different positions. For a deeper investigation on the cosmological implications of this approach, see Luminet (2001). In the Fig. 19, the coincident points along the time axis are linked by straight lines:

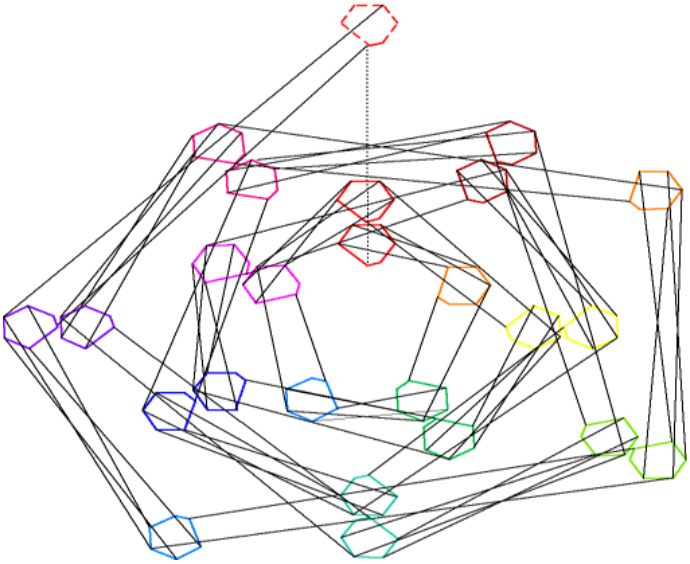


Fig. 19

Fig. n. 20, based on module 4-3, the chord (0,1,3,5,7,9) transposed upon levels with alternatively 4 and 2 common-notes, originates two canons, at 4 and 2-voices, with entries at ascending perfect fifth (See Ex. n. 4)

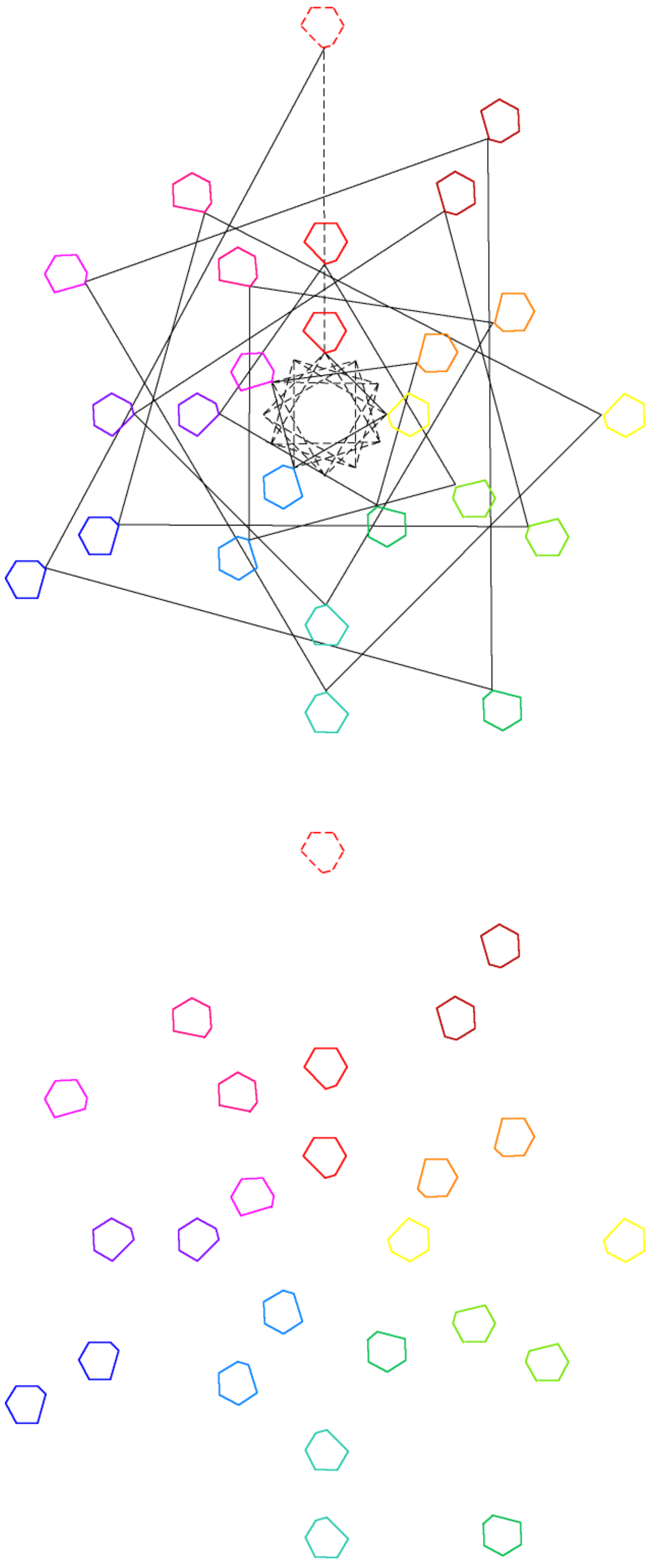


Fig. 20

Es. 4

- AUDIO sample 4 -

The symmetry of the kaleidocycle allows to reverse all intervals without any change of the structure (Ex. n. 5)

Es. 5

- AUDIO sample 5 -

It is also possible to superimpose a kaleidocycle to its inverse. One may notice that the tritone is the beginning and the end is the octave, and that there are never unison.

- AUDIO sample 6 -

In the three following figures (Fig. n.21) is the graphic transposition of the original kaleidocycle, of the reversed one and of the superimposition of both:

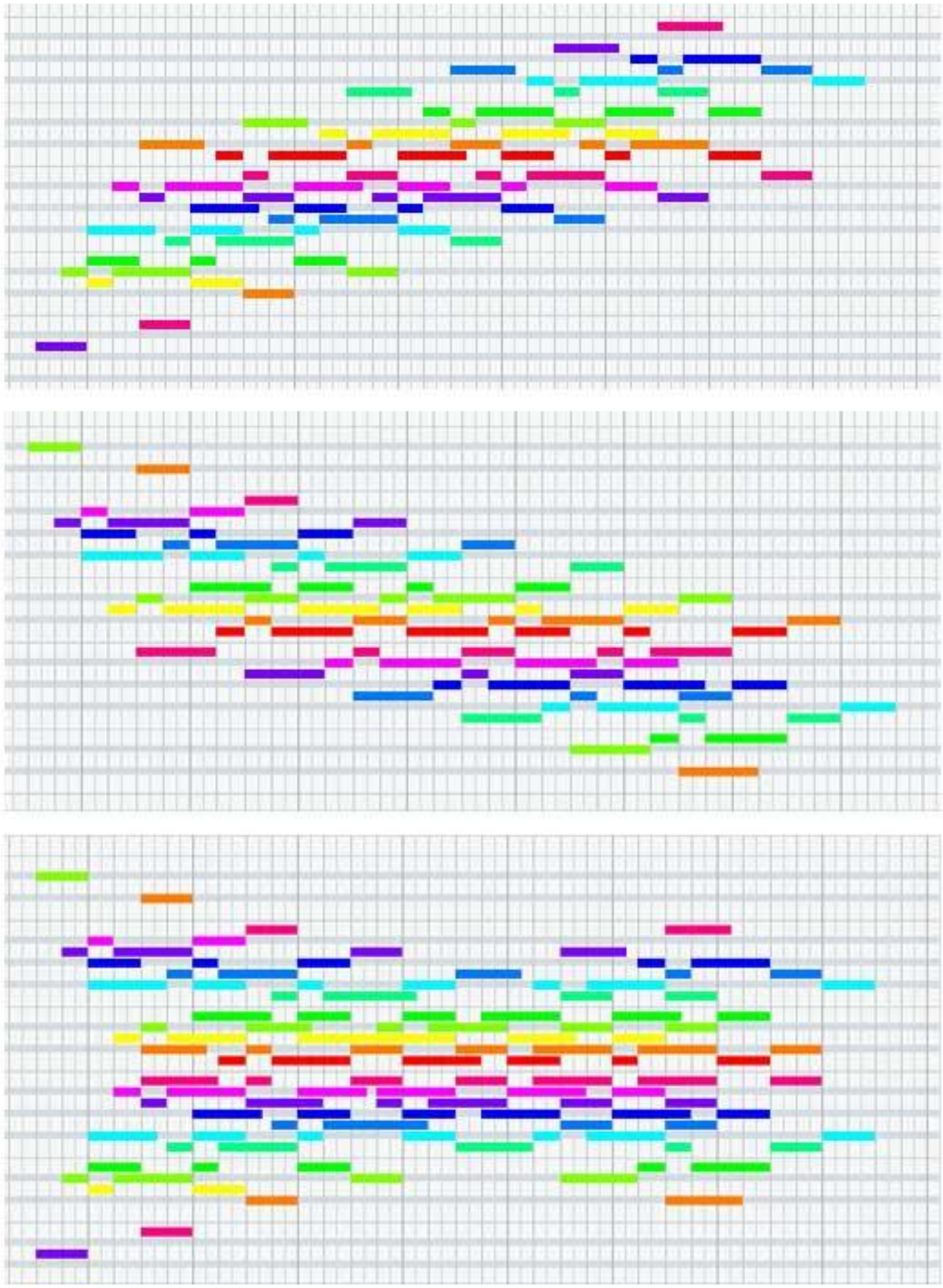


Fig. 21

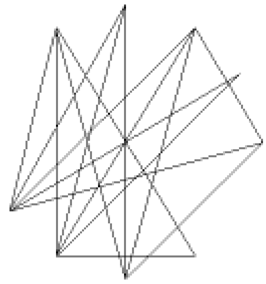
A kaleidocycle based on one chord can therefore be superimposed to another one derived from a different chord related to the former. It will thus have two complementary kaleidocycles related to a super-chord that includes both, so that there are never simultaneous repeats of the same pitch. It is also possible to have three or even four related kaleidocycles, according to the cardinal number of each supplementary chord [Verdi 2007, 154-183].
The relations among all components of a kaleidocycle may also become extremely complex; therefore the rigour of the development may have extreme outcomes, so as to lead every element of the musical construction back to a single matrix. The possibilities to be explored in this field are infinite and it may be puzzling when relations become too many to be ruled. On the other hand, a good musical outcome may not be automatically taken for granted.
In conclusion, it seems to be possible to assert that the logic of a kaleidocyclical structure is in some relation between the cycle module and the interval constitution of the supplementary chord. A kaleidocycle in which these relationships are such that only one solution is possible, is a perfect kaleidocycle. In such a structure, the ideal union between space becoming time and viceversa is realized.

6. Analysis as starting point of kaleidocyclic composition. An example

It is now possible to observe, by means of a harmonic analysis of some short compositions by Aleksandr Skrjabin, that all the elements of the kaleidocyclic logic are evident in latent form in this composer's work: indeed, in late Skrjabin it is not unusual to see a strict planning of transposition levels in such a way as to create kaleidocyclic compositions.

In the following graphs, the first column shows the frequency of the relations among the roots of the chords used. The mark " –" refers to the type of relation between two chords, for example "1-" means that the semitone relation (1) appears once; "6---------------------" means that the tritone relation (6) appears 15 times and so on. The second column shows the hierarchy of the single transpositions of the chords themselves, which is calculated on the basis of frequency and length of each appearing transposition. The mark "=1/8" refers to the unit upon which the amount of recurrence of a chord is calculated (determined with as many marks; therefore "C-" means that a chord upon root C totally appears for 1/8, while "C sharp"---------- means that a chord upon root C sharp appears totally for 10/8). The third column shows the graph with the sequence of the various transpositions represented by segments, which link the vertexes of an imaginary dodecagon- chromatic scale: the more regular and symmetrical the shape is, the more kaleidocyclic the corresponding composition is. An example of roots highlighting and of their mutual links, as to the first page of Skrjabin's Etude op. 54 n. 6, is made in Ex. n.6, where both roots and relations between them in terms of semitones are defined.

1) Etude op. 56 n. 4

Relations: -= one relation	Roots: - = 1/8	
1-	C-	
2--	C#----------	
3-	D----	
4--	D#--	
5-----------	E	
6---------------	F---	
7	F#-----	
8--	G----------	
9---	G#------	
10	A	
11	Bb	
	B----	

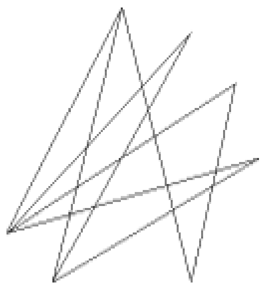
Relations: tritone (6) highly prevalent, but also perfect forth (5) is frequent.
Roots: predominance of tritone pairs (G-sharp, Gsharp-D, B-F).

Etude, Op. 56, No. 4 (1908)

Es. 6

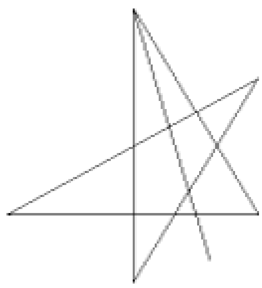
- AUDIO sample 7 -

2) Désir, op. 57, n. 1

Relations: - = one link	Fondamentali: - = 3/8	
	C----	
1	C#----------	
2	D-----------	
3-	D#-	
4	E	
5-----	F--	
6----	F#	
7----	G--------------------	
8--	G#---	
9	A	
10	Bb	
11	B	

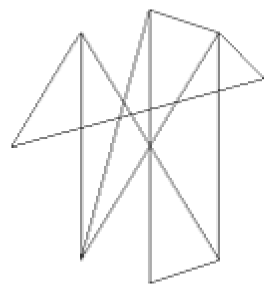
Relations: polarization upon fourth, fifth and triton.
Roots: frequent use of chords at fourth, fifth and tritone-interval.

3) Feuillet d'album op. 58

Roots: - = 3/4	Roots: - = 3/4	
	C------	
1	C#	
2	D--	
3	D#	
4	E-	
5--	F---	
6---	F#--------	
7--	G	
8--	G#--	
9-	A	
10	Bb	
11	B	

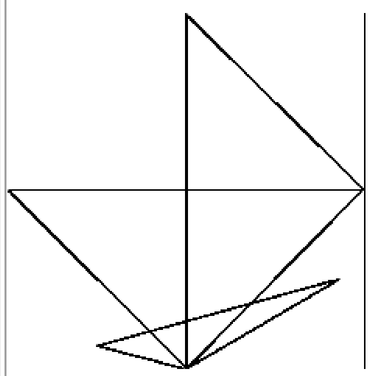
Relations: slightly highlighted tritone.
Roots: the F sharp-C polarization at tritone interval is eviden.

4) Poème op. 59 n. 1

Relations: - = one link	Roots: - = 6/8	
1 2 3 4 5---- 6----------------------- 7 8--- 9 10 11---	C---- C#--- D- D# E F----------- F#------- G---- G#- A Bb B--------	

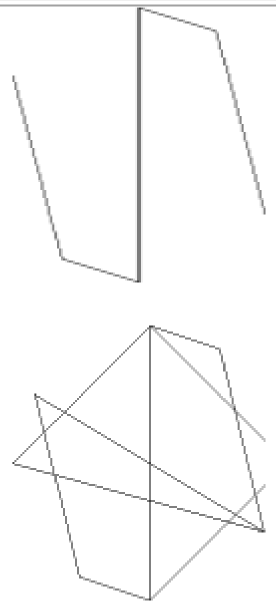
Relations: predominance of the tritone with items along a diminished-triad axis (F, A flat, B)
Roots: Most chords upon three tritones (F-B, G-Cis, C-F sharp). This piece is an example of Skrjabin's inclination to a kaleidocyclic logic.

5) Prelude op. 67 n. 1

Relations: - = one link	Roots: - = 2/8	
1 2 3----------- 4 5 6---------- 7 8 9-------- 10- 11-	C-------------------- C# D D#----------- E-------- F F#------------------------------------- G- G# A---- Bb B	

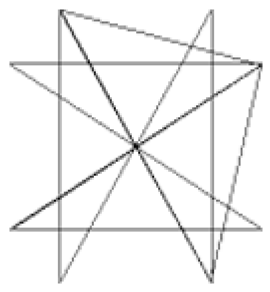
Relations: mainly along the minor-third axis
Roots: the chords upon C and Fsharp widely appear in almost all of the composition.

6) Poème op. 69 n. 1

Relations: - = one link	Roots: - = 3/4	
		
1	C--------	
2	C#--	
3--	D	
4	D#----	
5	E------	
6---------	F	
7	F#---	
8----	G--	
9-----	G#----	
10	A	
11--	Bb------	
	B	

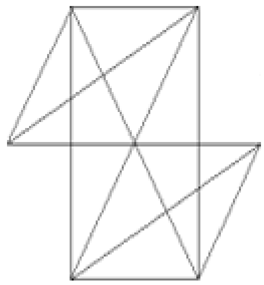
Here the logic of relations seems fading down, however the tritone is always evident. While the second picture shows all relations, the first one represents only the relations of the first section of the composition, pointing out its most evident connections and symmetries.

7) Poème op. 71 n. 2

Relations: - = one link	Roots: - = 3/4	
	C	
1	C#-	
2	D----------	
3--	D#	
4	E-	
5	F----------	
6----------------------	F#	
7	G-	
8-	G#----------	
9----	A	
10-	Bb	
11	B-----	

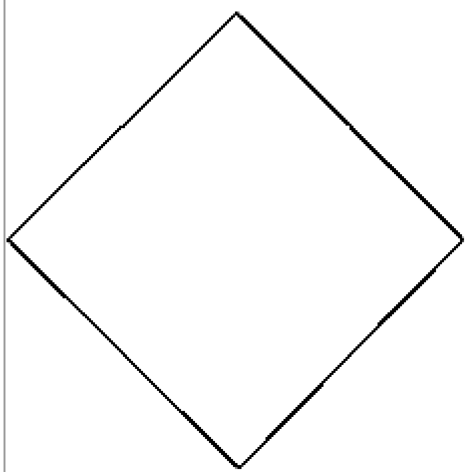
In this work, the predominance of a tritone relation is evident and it is reflected in the intensive use of minor-third intereval chords. This is indeed a projection into a kaleidocyclic logic. In the late works, Vers la flame op. 72, Two Poems op.73, 5 Preludes op. 74, this tendency is furthermore developed, since the tritone-relation gradually becomes a relation of a chord with itself.

8) Poème op. 73, n. 1

Relations: - = one link	Roots: - = 3/4	
1 2 3 4 5 6------------------------ 7 8------ 9 10 11	C C#-------- D D#-------------- E F----------- F# G-------- G# A------------- Bb B-----------	

The regularity and the symmetry of the pattern of this work points out that, in terms of music, repetitions and transpositions of chords develop a technique producing specific interval relations, planned in advance by the composer.

9) Prèlude op. 74 n. 3

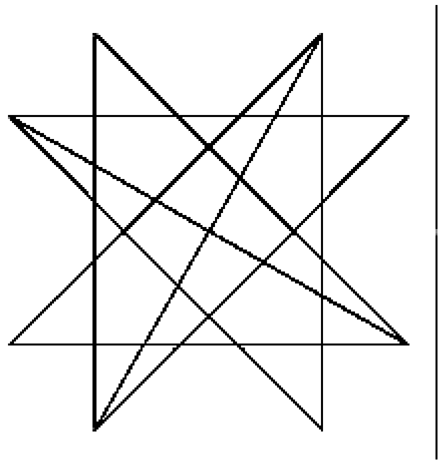
Relations: - = one link	Roots: - = 9/8	
1 2 3------ 4 5 6 7 8 9-- 10 11	C-------- C# C C#---- E F F#---------- G G# A---- Bb B	

The exclusive use of relations and chords along a minor-third axis is evident; Skrjabin had already used this technique in the introduction of Prometheus (Verdi 1996). This composer uses these relations, as well as tritone relations, very frequently, and they produce regular harmonic leadings which allow an evident "kaleidocyclic" interpretation.

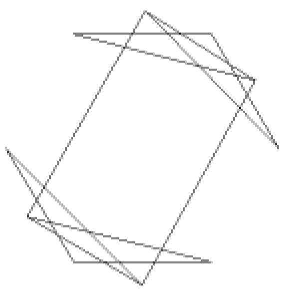
In my Cinque preludi-variazioni per pianoforte (1998), what has been pointed out in this analysis is developed furthermore: the rules of the cyclic relation are applied, so that the sorting harmonic figures become regular and symmetrical. The following examples highlight the overall framework of the relations used in the first, in the second and in the fourth pieces. In the first piece, the regularity of the relations (third, fifth and tritone) and of the frequency of the chords reflects a prearranged plan; in the second piece, it is worth emphasizing the strict cyclicity of frequency in the roots of the chord, whereas in the fourth piece, a different type of recurrence in frequency and sequence of chords must be highlighted.

1) Luigi Verdi, Cinque preludi-variazioni, n. 1

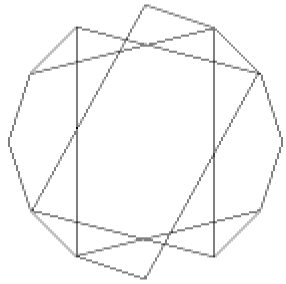
- AUDIO sample 8 -

Relations: - = one link	Roots: - = 2/4	
1	C	
2	C#---------	
3	D---	
4	D#	
5	E---------	
6-------	F---	
7----	F#	
8----	G---------	
9	G#---	
10	A	
11	Bb-----------	
	B---	

2) n. 2

Relations: - = one link	Roots: - = 3/8	
1	C----	
2-------------	C#----	
3	D----------------	
4---	D#--	
5	E	
6	F--	
7	F#----	
8	G----	
9------	G#---------------	
10	A--	
11	Bb	
	B--	

3) n. 4

Collegamenti: - = un collegamento	Roots: - = 3/8	
1---------- 2 3---- 4---- 5 6 7 8 9 10 11-	C- C#---- D---- D#- E--- F--------- F#- G---- G#---- A- Bb--- B---------	

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